APPENDIX - THE EQUIVALENCE THEOREM

The equivalence theorem states that the game result obtained by area counting is the same as the game result obtained by territory counting.

This proof is based on the proof given in the French rules (inspired by the AGA rules). The proof keeps track of stones and territories, and relies on rule 11 which ensures that Black and White made exactly the same number of moves. Assume the game has ended without dispute, and all prisoners removed.

BLACK COUNT	WHITE COUNT	MEANING
T_b	T_w	Territory
S_b	S_w	Stones on the board
P_b	P_w	Prisoners in opponent's lid
R_b	R_w	Pass stones given by player
$M_b = S_b + P_b + R_b$	$M_w = S_w + P_w + R_w$	Total moves
$A_b = S_b + T_b$	$A_w = S_w + T_w$	Area Score
$Q_b = T_b - P_b - R_b$	$Q_w = T_w - P_w - R_w$	Territory score

Table of Counts

Rule 11 ensures that the total moves by Black equals the total moves by White, so:

$$M_b = M_w$$

From the third last line of the table we then get:

$$S_b + P_b + R_b = S_w + P_w + R_w.$$

Hence the difference in the stone count is:

$$S_b - S_w = P_w - P_b + R_w - R_b$$

Now the area score difference is obtained from the second last line of the table as:

$$A_b - A_w = S_b - S_w + T_b - T_w.$$

We can eliminate the stone count difference in this expression to get:

$$A_b - A_w = P_w - P_b + R_w - R_b + T_b - T_w.$$

But from the last line of the table we see that the territory score difference is:

$$Q_b - Q_w = (T_b - P_b - R_b) - (T_w - P_w - R_w)$$

This can be re-arranged to give:

$$Q_b - Q_w = T_b - T_w + P_w - P_b + R_w - R_b$$

This is identical to the expression for the area score difference.

Corollary 1 Jigo

On a board with an odd number N of intersections, komi should be odd to allow jigo.

Suppose there are no seki positions, so every empty intersection is territory.

Then $A_b + A_w = N$, and the area score difference is $A_w - A_b = N - 2A_b$. The game score difference with a komi of K is:

$$D = A_w - A_b + K = N + K - 2A_b.$$

For D to be zero, N+K must be even. Since N is odd, we require K to be odd.

Corollary 2 Optimum Komi

On a board with an odd number of intersections, the minimum winning White score difference is obtained with fractional odd Komi.

Using the same notation as in Corollary 1, we have $D = N + K - 2 A_b$, and we will express $N=2 n+1$. We have established in Corollary 1 that the minimum possible game score difference is zero when komi is integral odd. For the minimum non-zero score difference, all we need to do then is to examine the cases of fractional even komi and fractional odd komi. Without loss of generality we can take the fractional part to be ½.

Suppose first that komi is fractional even, so $K = 2 k + ½$. The score difference D then has the value:

$$D = 2 (n+k-A_b) + 1+ ½.$$

The minimum winning value of D occurs when the factor multiplying 2 is zero (If the factor were -1, then D becomes a loss). In this case then the minimum winning value of D is $D_{min} =1½$.

Suppose next that komi is fractional odd, so K has the form $K = 2 k+1 + ½$. Then

$$D = 2 (n+k+1-A_b) + ½.$$

The minimum possible winning value of D is therefore $D_{min} = ½$.

This shows that fractional odd komi gives the lowest possible winning margin.

It is of interest to see how the minimum possible score differences vary for increasing values of komi:

Komi	0	½	1	1½
Minimum score difference	1	1½	0	½

The minimum score differences just repeat the above values as K is increased by 2 for any column in the table.

Corollary 3 Handicap games

The equivalence theorem holds for handicap games.

Suppose the handicap is H stones taken by Black. Then at move 1, Black places H stones on the board to start with, i.e. H-1 more stones than in an even game. Using the same notation as in the Table of Counts above, it follows that the number of moves by Black is given by $M_b = S_b+ P_b+ R_b - (H-1)$.

From the table, the number of moves by White is $M_w = S_w+ P_w+ R_w$. Since $M_b=M_w$, the stone count difference is given by:

$$S_b - S_w = P_w - P_b + R_w - R_b + (H-1)$$

Now by Rule 4 in a handicap game, White is given an additional H-1 points, so the area score difference is:

$$A_b - A_w = S_b - S_w + T_b - T_w.- (H-1) = P_w - P_b + R_w - R_b + T_b - T_w.$$

This is identical to the expression for the territory score difference:

$$Q_b - Q_w = T_b - T_w + P_w - P_b + R_w - R_b$$